Flat Fading Channel Envelope Prediction and Equalization

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Abstract

Fading envelopes generated under constant velocity and constant acceleration conditions, modeling mobile communication scenarios, are predicted with a time-varying recursive linear predictor. The envelope prediction is used in an automatic gain controller with the idea of compensating for the amplitude distortion of a flat fading channel. Assuming that the difference between the phase distortions of two consecutive symbols is very small, the effect of the phase distortion is removed using a differential modulation scheme. Simulations show that, with fading envelope compensation, highly bandwidth efficient modulation schemes such as QAM can be used even in a flat fading environment.

1. Introduction

A linear predictor requires a priori knowledge of the *auto-correlation function* (ACF) or the correlation matrix of the colored input [1]. When the predictor coefficients are kept fixed, it is not expected to perform well in a time varying situation. Therefore a recursive linear envelope predictor is suggested here, to adaptively and recursively track the correlation structure of the faded signal envelope.

As the predictor coefficients are updated for every input sample, the recursive linear predictor (RLP) is expected to perform better when the correlation properties of the input change with time. To make the RLP robust in the time-varying case, a forgetting factor – as in the RLS or *Windowed Recursive Least Squares* (WRLS) [2] algorithm – will be used while updating the correlation sequence estimate. The RLP predicted envelope of the complex flat fading channel is then used to mitigate the effect of the fading channel on a QAM modulated signal.

2. One Step Ahead Predictor

The output $\hat{x}(n)$, of a one step ahead predictor, can be written in terms of the past M values of input, $\mathbf{x}=[x(n-1) \quad x(n-2) \quad \cdots \quad x(n-M)]^T$ as:

$$\begin{aligned}\hat{x}(n) &= -\sum_{i=1}^{M} a_i^M x(n-i) \\ &= -\mathbf{a}^T \mathbf{x}\end{aligned} \tag{1}$$

where $-\mathbf{a}=[-a_1^M \quad -a_2^M \quad \ldots \quad -a_M^M]^T$ is the vector of predictor coefficients. The normal equations for the *one step ahead predictor* are [1]:

$$2\left[r_{xx}(l)+\sum_{k=1}^{M} a_k^M r_{xx}(k-l)\right]=0\,,\ l=1,2,\ldots,L \tag{2}$$

In matrix form,

$$\mathbf{Ra}=\mathbf{b} \tag{3}$$

where $\mathbf{R}$ in (3) is the auto-correlation matrix, defined as:

$$\mathbf{R}=\begin{bmatrix} r_{xx}(0) & r_{xx}(1) & \cdots & r_{xx}(M-1) \\ r_{xx}^*(1) & r_{xx}(0) & \cdots & r_{xx}(M-2) \\ \vdots & \vdots & \cdots & \vdots \\ r_{xx}^*(L-1) & r_{xx}^*(L-2) & \cdots & r_{xx}^*(L-M) \end{bmatrix} \tag{4}$$

and vector $\mathbf{b}$ is also auto-correlation sequence based:

$$\mathbf{b}=-[r_{xx}(1) \quad r_{xx}(2) \quad \cdots \quad r_{xx}(L)]^T \tag{5}$$

For $L>M$, the system of normal equations is over-determined (the pseudo-inverse of $\mathbf{R}$ can be used to find a least squares solution to (3)), generally yielding better results for noisy input signals.

3. Recursive Linear Predictor

In the absence of a priori knowledge, the *auto-correlation function* (ACF) of the input sequence to the predictor needs to be estimated. For a signal with time-varying statistical properties, the ACF needs to be estimated for the present conditions, so that the far past data values need to be discounted. Thus, a window-based technique is used for recursive ACF estimation.

At any instant the ACF is estimated by using the present data and the past ACF estimate. A window of length N is used to consider only the most recent N samples of input data, denoted $\mathbf{x}_N(n)$, from which an N length instantaneous auto-correlation (single-sided) estimate $\hat{\mathbf{r}}_i(n)$ is calculated as follows:

$$\begin{aligned} &\Theta = \mathrm{DFT}_{2N}\{\mathbf{x}_N(n)\} \\ &\hat{r}_i(p) = \mathrm{IDFT}_{2N}\{\Theta.*\Theta^*\}/N, \quad p = 0,1,\cdots,N-1 \\ &\hat{\mathbf{r}}_i(n) = [\hat{r}_i(0) \quad \hat{r}_i(1) \quad \cdots \quad \hat{r}_i(N-1)]^T \end{aligned} \tag{6}$$

The ACF estimate is then recursively updated:

$$\hat{\mathbf{r}}(n) = \frac{\lambda\,\hat{\mathbf{r}}(n-1) + \hat{\mathbf{r}}_i(n)}{\lambda + 1}, \quad 0 << \lambda \leq 1 \tag{7}$$

In (7), $\hat{\mathbf{r}}(n)$ is the estimate of the ACF at instant n, and λ is the forgetting factor, as in the well-known exponentially weighted RLS algorithm. The infinite length exponentially decaying window (for λ<1) is shown in Fig. 1. The exponential window not only has an infinitely long tail, it either penalizes near present data too much, or discards old information too slowly. The latter limits the utility of RLS type forgetting in a non-stationary environment.

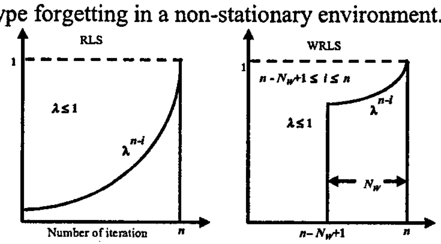


Fig. 1: RLS and WRLS Window Functions.

To improve performance, a rectangular window of length N_W is used to truncate the exponential window so that the far enough past is discarded completely. The RLS alternative is known as the *Windowed Recursive Least Squares* (WRLS) algorithm [2]. The window function of WRLS is shown in Fig. 1. The WRLS algorithm performs better than RLS for non-stationary signal estimation [2]. Using WRLS, the update in (7) becomes:

$$\hat{\mathbf{r}}(n) = \frac{\lambda\,\hat{\mathbf{r}}(n-1) + \hat{\mathbf{r}}_i(n) - \lambda^{N_W}\,\hat{\mathbf{r}}(n-N_W)}{\lambda + 1 - \lambda^{N_W}} \tag{8}$$

Since the window used to collect the input data is of length N, at the beginning of the estimation process it is implicitly assumed that the correlation property of the input sequence will not be the same after N input samples. Therefore, while updating the correlation sequence $\hat{\mathbf{r}}(n)$, the length of the rectangular window N_W should reasonably be less than or equal to the length of the windowed input sequence $\mathbf{x}_N(n)$, i.e.,

$$N_W \leq N \tag{9}$$

Using WRLS, which completely discards the far enough past, a higher value of the forgetting factor λ can be used to give more weight to the immediate past samples. The ACF estimate $\hat{\mathbf{r}}(n)$ in (7) or (8) is then used to form $\hat{\mathbf{R}}(n)$ and $\hat{\mathbf{b}}(n)$, and the predictor coefficients for time instant n are found by solving (3).

4. Prediction of the Fading Envelope

The fading envelope of a flat fading channel is now predicted by using the recursive linear predictor. The power spectrum of a flat fading envelope can be expressed in terms of the carrier frequency f_c and f_m, as follows [3]:

$$S_{E_C}(f) = \begin{cases} \dfrac{1.5}{\pi f_m \sqrt{1 - \left(\dfrac{f - f_c}{f_m}\right)^2}}, & |f - f_c| \leq f_m \\ 0, & |f - f_c| > f_m \end{cases} \tag{10a}$$

The baseband power spectrum follows from f_c=0.

$$S_{E_B}(f) = \begin{cases} \dfrac{1.5}{\pi \sqrt{f_m^2 - f^2}}, & |f| \leq f_m \\ 0, & |f| > f_m \end{cases} \tag{10b}$$

The non-white *power spectral density* (PSD) of the fading envelope implies that its time samples are correlated. The correlation is stronger for smaller f_m or higher sampling frequency f_s. Therefore, if the fading envelope is sampled at a rate much higher than $2f_m$, future values of the fading envelope can be predicted reasonably well from its present and near past samples [4]. A simple one-step linear predictor can be used.

The predictor described in Sections 2 and 3 was used to predict a Rayleigh fading envelope generated with constant velocity and constant acceleration. Clark's model [5] is an efficient frequency domain approach for implementing the fading envelope with constant velocity. However, it can not be used for generating the fading envelope with constant acceleration, so that the general time domain model [5] was extended for that case [6].

4.1. Fading Envelope with Constant Velocity

Since the maximum Doppler spread f_m of the fading envelope is constant for a constant velocity, f_m will be used as the independent variable for all results. A Rayleigh fading envelope with f_m=100 Hz and f_s=8 kHz was generated and shown in Fig. 2 together with its prediction.

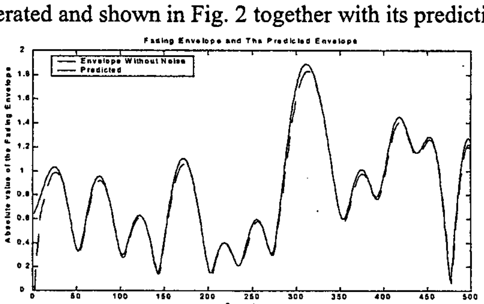
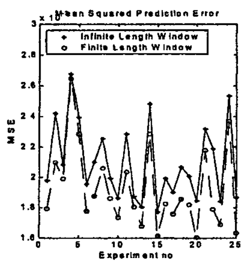


Fig.2: Actual and Predicted Fading Envelope. RLS Type Window Used to Update the Correlation Sequence.

A length 7 RLP was used, and the ACF estimate updated using an RLS type window with λ=0.9. The **R** matrix was 20x7, and the input window length $N = 50$.

Using WRLS ACF updating, with N_W=50, produced almost the same result (not shown). A comparative performance analysis between the two approaches is presented on the basis of the envelope prediction error. The time average Mean Squared Prediction Error (MSPE) is used for comparison:

$$MSPE = \frac{1}{N}\sum_{n=0}^{N-1} e^2(n) = \frac{1}{N}\sum_{n=0}^{N-1}\left(x(n) - \hat{x}(n)\right)^2 \qquad (11)$$



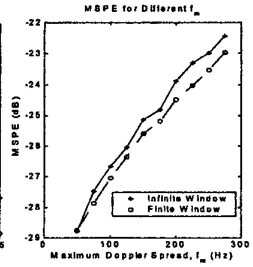


Fig. 3: MSPE for RLS and WRLS Windows for Different Realizations (f_m=100 Hz).

Fig. 4: Mean Squared Prediction Error for Different f_m.

The MSPE for the two approaches are shown in Fig. 3, where – to avoid transient effects – the first 50 samples were not used in the calculation. For the same Doppler spread, the MSPEs are different for different fading envelope realizations. Therefore, to get an estimate of MSPE, the simulation was run 50 times, with the same realization used for the differently windowed estimate updates. Fig. 4 shows the results for both approaches, for different values of maximum Doppler spread f_m.

From Fig. 4 we see that the prediction error increases with an increase in f_m. For higher mobile velocity the randomness of the fading envelope increases, resulting in reduced correlation among the samples of the fading envelope and increased prediction error. Fig. 4 also shows that for any f_m, the RLP performs best when the finite length WRLS approach is used.

4.2. Fading Envelope with Constant Acceleration

To reduce the number of samples, and consequently processing time, a lower sampling frequency (f_s=2000 Hz) was used in this simulation. Mobile velocity changed from 30 to 58 mph in 2.8 seconds, for an acceleration of 10 miles/sec^2. A portion of the envelope is shown in Fig. 5, together with the output of the RLP based on WRLS updating. The simulation used $N_W = 15$ and $\lambda = 0.95$. Ten different realizations were used to estimate the MSPE value for a particular acceleration. We similarly evaluate MSPE for RLS based updating, which used λ=0.9.

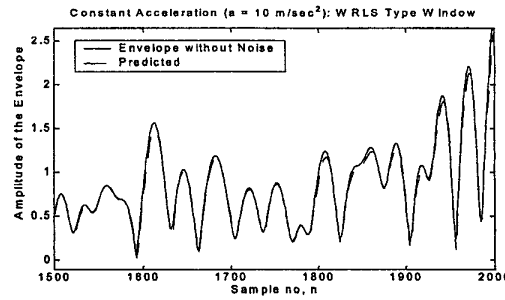


Fig. 5: Rayleigh Fading and Predicted Envelope for 10 miles/sec² Constant Acceleration – WRLS Type Window Used in Predictor & Sampling Frequency f_s = 2 kHz.

The MSPE values for different accelerations are shown in Fig. 6, for both RLS and WRLS updating of the ACF.

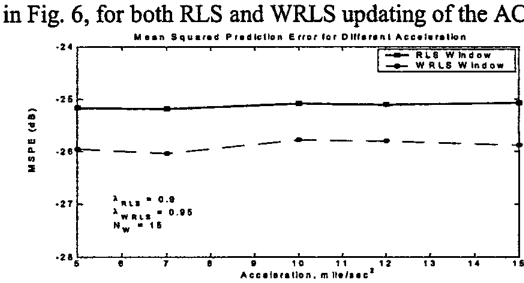


Fig. 6: MSPE for Different Accelerations.

From Fig. 6 we observe that, unlike in the constant velocity case, RLP performance (in terms of MSPE), based on either WRLS or RLS updating, does not degrade with an increase in the acceleration. Fig. 6 also shows that WRLS RLP performs approximately 1 dB better than RLS RLP when mobile velocity changes with constant acceleration. This performance improvement comes in a trade-off with increased memory requirements for WRLS relative to RLS. Assuming that WRLS does not provide sufficient performance improvement to offset its increased memory requirements, the infinite length (RLS) window is used in the next section to reduce complexity.

5. QAM Over the Flat Fading Channel

In a flat fading channel the transmitted signal is distorted in both amplitude and phase, as shown below:

$$\begin{aligned} r(n) &= s(n) * a_f(n)e^{j\theta_f(n)} + w(n) \\ &= \left\{a(n)e^{j\theta(n)}\right\} * a_f(n)e^{j\theta_f(n)} + w(n) \qquad (12) \\ &= a(n)a_f(n)e^{j\{\theta(n)+\theta_f(n)\}} + w(n) \end{aligned}$$

In equation (12), $a(n)$ and $\theta(n)$ are the amplitude and phase of the information signal, respectively, and $a_f(n)$ and $\theta_f(n)$ correspondingly for the fading envelope, while $w(n)$ is white noise with variance σ_w^2. If the amplitude distortion $a_f(n)$ follows Rayleigh statistics (as shown in

Fig. 2 or 5), the phase distortion $\theta_f(n)$ follows a uniform distribution over $[0,2\pi]$ radians [5]. It was shown in Section 4 that the distortion in amplitude can be predicted, and therefore used in an *Automatic Gain Controller* to compensate for the effect of the amplitude distortion.

Again, the phase distortion is not a white process since it results from two orthogonal and colored Gaussian sequences. Assuming that the difference between the phase distortions at two consecutive symbols is very small, the effect of the phase distortion can be removed by using a differential modulation scheme [5]. Let us consider a QAM signal with instantaneous amplitude $a(n)$ and instantaneous phase $\phi(n)$, so that

$$x_{QAM}(n) = a(n)e^{j\phi(n)} \tag{13}$$

In a differential QAM (DQAM) scheme the instantaneous amplitude is the same as for the original QAM signal but the instantaneous transmitter phase $\theta(n)$ as follows.

$$\begin{aligned} x_{DQAM}(n) &= a(n)e^{j\theta(n)} \\ \theta(n) &= \theta(n-1)+\phi(n) \end{aligned} \tag{14}$$

Fig. 7 shows the QAM and DQAM signal constellations.

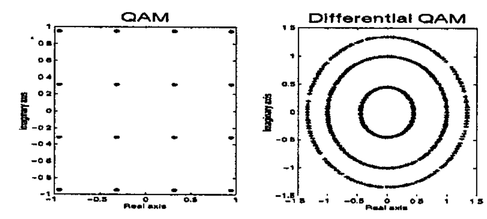


Fig. 7: Signal Constellations for QAM and DQAM.

If transmitted through a flat fading (e.g., complex Rayleigh fading) channel, the DQAM constellation will be distorted and, as seen from (12), retrieving information from the received signal becomes quite impossible because of the amplitude distortion. To recover the information a receiver is proposed in which the amplitude distortion is compensated via RLP and the phase distortion is removed using usual differential detection techniques. Fig. 8 shows the block diagram of the proposed receiver.

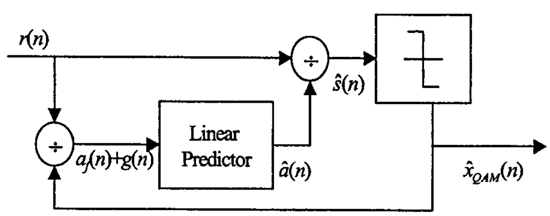


Fig. 8: Proposed Receiver Model.

The receiver exploits the coloration [5] of the fading envelope to predict its amplitude. To estimate the amplitude $a_f(n)$ of the fading envelope, first a signal is required that can be represented as $a_f(n)+g(n)$, where $g(n)$ is a white signal with variance σ_g^2. The receiver generates this signal using decision feedback as shown in Fig. 8. For simplicity it will be assumed that the amplitude $a(n)$ of the information signal and its estimated value, the output of the hard-limiter, are equal. The white noise $g(n)$ getting into the predictor, can be represented as follows

$$\begin{aligned} g(n) &= \frac{w(n)}{\hat{a}(n)}e^{-j\{\theta(n)+\theta_f(n)\}} = \frac{w_1(n)}{\hat{a}(n)} \\ \hat{x}_{DQAM}(n) &= \hat{a}(n)e^{j\{\theta(n)+\theta_f(n)\}} \end{aligned} \tag{15}$$

In (15), $w_1(n)$ is a complex white Gaussian noise with variance σ_w^2, and $\hat{a}(n)$ is the amplitude of the DQAM signal – to be used in the demodulation process – which has the following probability mass function.

$$f_{\hat{A}}(\hat{a}) = \sum_{i=1}^{3} p_i\delta(\hat{a}_i) \tag{16}$$

Since differential encoding does not change the amplitude distribution of the QAM signal, (16) can be expanded as:

$$f_{\hat{A}}(\hat{a}) = \frac{1}{4}\delta(\hat{a}-\sqrt{2}) + \frac{2}{4}\delta(\hat{a}-\sqrt{10}) + \frac{1}{4}\delta(\hat{a}-\sqrt{18}) \tag{17}$$

In (17), the average power of the QAM signal is assumed to be 10. Using (15) and (17), it can be concluded that $g(n)$ is white with the following distribution:

$$G: \sum_{i=1}^{3} p_i N\left(0, \frac{\sigma_w^2}{a_i^2}\right) \tag{18}$$

It can be shown that $g(n)$ is a zero mean process with variance σ_g^2, where

$$\sigma_g^2 = \sigma_w^2 \sum_{i=1}^{3} \frac{p_i}{a_i^2} \tag{19}$$

The predictor estimates the value of $a_f(n)$ with some error, which depends on the following parameters:

1. Presence of $g(n)$ and the forward prediction error, the effect of which is shown in [7];
2. Effect of non-stationarity, presented in Section 4;
3. Imperfect prediction filter order estimation.

The envelope estimate is used to compensate for the amplitude distortion introduced by fading, as shown in Fig. 8. The amplitude-compensated received signal is then passed through a hard-limiter to determine the nearest amplitude level, after which a differential detector is used to remove the phase distortion caused by fading and to recover the transmitted information.

6. Simulation Results

Simulations were performed to determine the performance of the proposed receiver under different channel conditions with the following parameters: $M = 7$; $\lambda = 0.9$; $N_W = 50$. A training sequence of length 50 was used at the beginning of the information sequence to provide the RLP enough time to fix its coefficients. Fig. 9 shows the received signal, having been subject to fading.

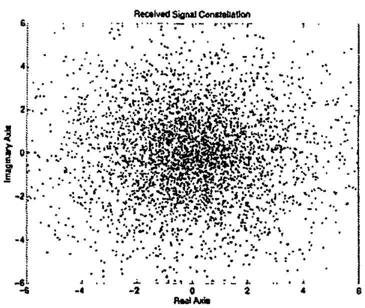


Fig. 9: The Received Signal $r(n)$, for f_m = 50, Es/No =25 dB.

Fig. 10 shows the constellation after envelope equalization, using RLP of the fading envelope.

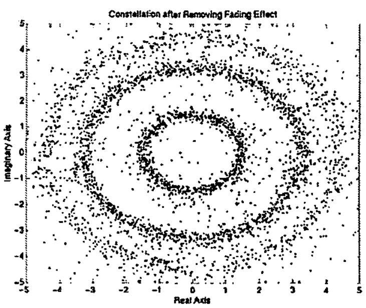


Fig. 10: Constellation after Envelope Equalization.

Fig. 11 shows the very recognizable constellation after differential demodulation.

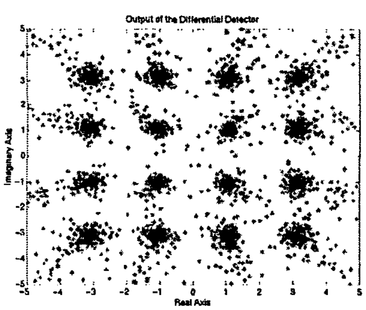


Fig. 11: Constellation after Differential Demodulation.

Fig. 12 shows more extensive simulation results to give an idea of the performance (BER) of the proposed receiver. The reasons for deviations between the BER curves of the proposed and regular DQAM receivers are twofold. Coefficient mis-adjustment error, due to the presence of external noise $g(n)$, degrades the receiver even in the absence of fading (*DQAM with receiver* in Figure 12). Also, as shown in Section 4, the predictor cannot perform perfectly in the presence of fading. This imperfection depends on the Doppler frequency f_m. Fig. 4 shows that this error is around –25 dB for f_m =150 Hz. This extra noise explains the performance deviation of the receiver in the presence of fading. Moreover, due to this error, the BER curves become horizontal beyond the SNR where external noise is negligible relative to the prediction error.

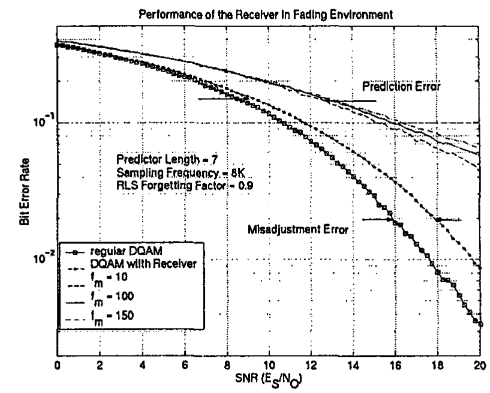


Fig. 12: Performance (BER) of Proposed Receiver.

As presented here, the system does not use any error correction coding. If a rate ½ convolution encoder were added, the BER performance is expected to improve by around 6 dB. An interleaver can be used to avoid the burst errors caused by differential decoding.

7. Conclusion

Recursive linear prediction of the flat fading envelope, either with finite or infinite length window, can be used to provide *automatic gain control* compensation for the fading effect in a channel. Simulations show that, with compensation for the fading envelope, highly bandwidth efficient modulation schemes such as QAM can be used even in a flat fading environment.

8. References

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