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High-Resolution Spectral Estimation for Twisted-Pair Length Estimation

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Abstract

DSL access over a POTS local loop requires accurate characterization of that loop. One approach to this problem is to identify the loop structure first and then simulate and analyze the identified loop. The most important task in the identification process is to estimate the length of twisted-pair segments in local loops. We present an innovative approach based on the loop frequency response. The procedure exploits the assumption that the loop frequency response is a linear combination of complex sinusoids, utilizing the MODE-Type algorithm (Cedervall, Stoica & Moses, 1995). Numerical simulation results are included to illustrate the accuracy that can be achieved.

1. Introduction

One of the limiting factors in providing high-speed data services—such as ADSL (asymmetric digital subscriber line)—over the plain old telephone system (POTS) is the physical distance and configuration of the network between a subscriber and the service provider. These services are deployed over existing twisted pair copper loops. Long twisted-pair copper loops, however, cannot sustain high-speed data communication due to dispersion. Hence, for the service provider, it is crucial to assess the configuration of the existing twisted-pair network so that the possible service data rate can be predicted.

We present the novel application of a high-resolution spectral analysis technique to twisted-pair length estimation. The estimation is based on a single-ended frequency response measurement (or reflectometry measurement) of a loop. Our procedure exploits the knowledge that the single-ended loop frequency response is an infinite sum of frequency responses, each characterizing one particular reflection path due to discontinuities. This is deduced from the fact that the single-ended frequency response directly relates to the time-domain reflectometry (TDR) response. The TDR technique sends an electrical pulse into a cable and—at the same end—observes the reflections. The length of the cable (or distances to discontinuities along the pulse path) relates to half the time at which the corresponding reflection returns to the measurement end. This method, however, is not very effective with twisted-pair cable because of its highly dispersive nature. Weaker reflections can easily be buried by stronger dispersed reflections (Ikuma, 2001).

A high-resolution spectral analysis technique, namely the Method Of Direction Estimation Type (MODE-Type) algorithm, is applied to overcome the dispersive nature of the twisted-pair cables. The MODE-Type algorithm is a subspace-based algorithm that resolves a finite sum of complex sinusoids and estimates parameters of these cisoids. The signal model for the MODE-Type algorithm is

$$\tilde{y}_k = \sum_{l=1}^{L} a_l \rho_l^k + \tilde{e}_k, \quad k = 1, 2, \ldots, N \tag{1.1}$$

where $\tilde{y}_k$ is the observed signal; $a_l \in \mathbb{C}$ and $\rho_l \in \mathbb{C}$ are scaling and damping factors, respectively; N is the number of data samples; and $\tilde{e}_k$ is an additive circularly symmetric complex white Gaussian noise. The twisted-pair reflection frequency response model closely follows (1.1) over a limited frequency range, as shown below. The dispersion of the twisted-pair is modeled with a damped sinusoid (i.e., $|a_l| < 1$). Hence, the MODE-type algorithm can be employed to decompose the loop frequency response into a sum of dominant reflection frequency responses. The group delay of the estimated reflection frequency response is shown to be affinely related to the length (or the total path distance to the corresponding discontinuity).

2. Twisted-Pair Reflection Frequency Responses

FIGURE 1 shows the typical reflectometry measurement setup where V_g and Z_g indicate source voltage and impedance, respectively; and V_o represents the measured voltage.

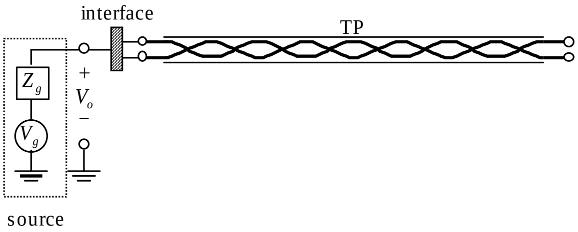

FIGURE 1. Reflectometry measurement setup.

The other end of the TP is modeled by an open circuit. This measurement setup can be expressed as a linear time-invariant (LTI) system with input signal $v_g(t)$ and output signal $v_o(t)$. The frequency response of this LTI system is defined by

$$H(f) = \frac{V_o(f)}{V_g(f)} \tag{2.1}$$

Based on transmission line theory (Chen, 1989), the frequency response in (2.1) can be expressed as an infinite sum of "reflection" frequency responses $\{H_p(f)\}_{p=0:\infty}$:

$$H(f) = H_0(f) + \sum_{p=1}^{\infty} H_p(f) \tag{2.2}$$

The first term $H_0(f)$ is related to $v_g(t)$ seen at the measurement interface, evaluated by

$$H_0(f) = \frac{Z_o(f)}{Z_o(f) + Z_g} \tag{2.3}$$

with $Z_0(f)$ being the characteristic impedance of the twisted-pair loop. The remaining reflection frequency responses are defined by

$$H_p(f) = H_0(f)\mathrm{T}(f)\Gamma^{p-1}(f)e^{-2pl\gamma(f)} \tag{2.4}$$

with line length *l*, propagation function $\gamma(f)$, reflection coefficient $\Gamma(f)$, and transmission coefficient $\mathrm{T}(f)$ at the boundary between the line and the measurement equipment.

A typical frequency response is illustrated in FIGURE 2a, and the corresponding first four reflection frequency responses are shown in FIGURE 2b. At higher frequencies (> 1 MHz) the reflection magnitude responses become log-affine (i.e., damped exponential in linear scale) over a short range of frequency. The phase responses are essentially linear (with slight DC offset). This observation leads to the *i*-th reflection frequency response model approximation (Ikuma, 2001):

$$H_p(f) \approx a_p \rho_p^f \tag{2.5}$$

over a narrow range of frequency. Hence, the signal model in (1.1) of the MODE-Type algorithm is an adequate approximation for the overall response in (2.2). Though the overall response theoretically contains infinitely many reflections, the reflection frequency responses corresponding to the terms associated with later reflections are much attenuated compared to the first few terms and become negligible numerically.

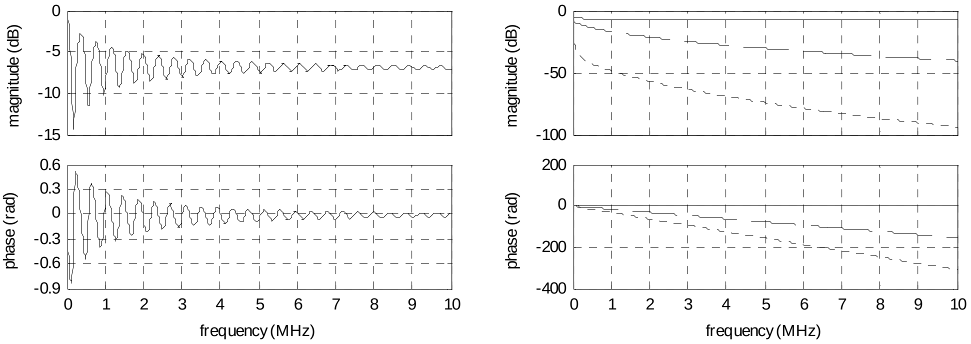

FIGURE 2. (a) Typical frequency response and (b) its first three reflection components.

3. Length Estimation Algorithm

Our twisted-pair length estimation procedure is implemented in five steps. The first two steps consist of data preprocessing (i.e., removal of known information) and the estimation of the number of resolvable terms. They are followed by the execution of the MODE-type algorithm, conversion of the MODE-type estimate to the length estimates of the twisted-pair segment, and finally selection of the best length estimate. In order to carry out the procedure, we assume that the characteristics of the twisted pair under investigation are completely known.. This includes $Z_0(f)$, $\gamma(f)$, $\Gamma(f)$, and $\mathrm{T}(f)$. Also, the source impedance Z_g is assumed known. The signal measurement used is a set of frequency response measurements, equi-spaced over the range that the MODE-Type model is applicable for. The *k*-th measurement is denoted as H_k, and its resolvable reflection components are denoted as $H_{k,p}$. Accordingly, the algorithm is based on the sampled verson of (2.2):

$$H_k = H_{0,k} + \sum_{p=1}^{\infty} H_{p,k} \quad \text{for } k = 0 : K-1 \tag{3.1}$$

1. Data Preprocessing. With the twisted-pair characteristics known, the unknown parameters in (2.3)–(2.4) are reduced to only the length parameter *l*. Moreover, the known information can be removed from (3.1). First, $H_{0,k}$ is entirely known and is subtracted from (3.1), producing the “residual” frequency response:

$$H_{e,k} = \sum_{p=1}^{\infty} H_{0,k} \mathrm{T}_k \Gamma_k^{p-1} e^{-2pl\gamma_k} \tag{3.2}$$

Furthermore, $H_{0,k}\mathrm{T}_k$ is common for all summation terms and thus can be removed from (3.2):

$$\tilde{H}_k \triangleq \sum_{p=1}^{\infty} \Gamma_k^{p-1} e^{-2pl\gamma_k} \tag{3.3}$$

2. Estimation of the Number of Strong Terms. To obtain reliable MODE-type estimates, it is important to obtain an accurate estimate of the number of dominant, resolvable reflection components in (3.3). The information theoretic criterion technique proposed by Wax and Kailath (1985) is used for this estimation. We denote this estimate to be $\hat{L}$. Typically, this estimator reports $\hat{L} = 1:3$, depending on the length of the segment.

3. MODE-Type Algorithm. The MODE-Type algorithm is applied to $\tilde{H}_k$ assuming $\hat{L}$ components. We denote the resulting MODE-Type estimates as $\{\hat{a}_l, \hat{\rho}_l\}$ for $l = 1:\hat{L}$.

4. MODE-Type Estimates to Length Estimates. Each MODE-Type estimate is converted to a length estimate $\hat{l}_l$. Comparing the *l*-th term of (estimated) (1.1) and the first term of (3.3) yields

$$\hat{a}_l \hat{\rho}_l^k \triangleq e^{-2\hat{l}_l \gamma_k} \tag{3.5}$$

The length is then obtained from the phase relationship:

$$\arg\left(\hat{a}_l \hat{\rho}_l^k\right) = \arg\left(e^{-j2\hat{l}_l \beta_k}\right) \tag{3.6}$$

where β_k is the phase function of the twisted-pair (the imaginary part of γ_k). Further algebraic operation yields

$$\arg(\hat{\alpha}_l) + k \arg(\hat{\rho}_l) = -2\hat{l}_l \beta_k \tag{3.7}$$

The twisted-pair phase function is known to be β_k affine (Ikuma, 2001). Thus, we can define

$$\beta_k \triangleq m_\beta k + c_\beta \tag{3.8}$$

Substituting (3.8) into (3.7) yields

$$\arg(\hat{\alpha}_l) + k \arg(\hat{\rho}_l) = -2\hat{l}_l \left(m_\beta k + c_\beta\right) \tag{3.9}$$

This implies that the length estimate can be obtained from either $\arg(\hat{\alpha}_l)$ or $\arg(\hat{\rho}_l)$. Since β_k tends to be close to linear (i.e., $c_\beta \approx 0$), we solve for $\hat{l}_l$ from $\arg(\hat{\rho}_l)$:

$$\hat{l}_l = -\frac{\arg(\hat{\rho}_l)}{2m_\beta} \tag{3.10}$$

5. Length Estimate Selection. Of the $\hat{L}$ length estimates obtained in Step 4, we can immediately discard any negative ones. The following minimization criterion is employed to determine the best length estimate:

$$J\left(\hat{l}_l\right) = \sum_{l=0}^{\hat{L}} \left| H_k - \hat{H}_k\left(\hat{l}_l\right) \right|^2 \tag{3.10}$$

4. Performance Evaluation

The presented simulation results are based on an analytical model for twisted pair cables (ANSI T1.601-1999). The model is used to generate the frequency response data to be processed. Also, the group-delay-to-length factor is predetermined from the model. The length of the twisted-pair is varied from 10 m to 10 km (34 cases). The observed data that we have used are 101 equally spaced samples of the TP frequency response over 1 MHz to 2 MHz. The effective sample spacing is 10 kHz. The MODE-Type algorithm is configured with snapshot length $2\hat{L}$. Snapshots are maximally overlapped (snapshot offset = 1 sample).

The estimates below are obtained from 20 independent simulations for selected segment lengths. The estimation results with clean measurements (no noise) are shown in FIGURE 3a. The algorithm breaks down at 5 km due to loss of numerical accuracy. Prior to breakdown, the estimation errors are within 4% of the actual lengths (less than 0.3% above 100 m). These errors are introduced by the approximation in (2.5). With the injection of additive complex Gaussian noise with variance $\sigma^2 = 10^{-4}$, the outcome is shown in FIGURE 3b. The resolution is reduced to ~700 m. The energy in the reflections diminishes exponentially as a function of length. Hence, the far reflections will be decisively masked by noise at some length, resulting in the observed algorithm breakdown.

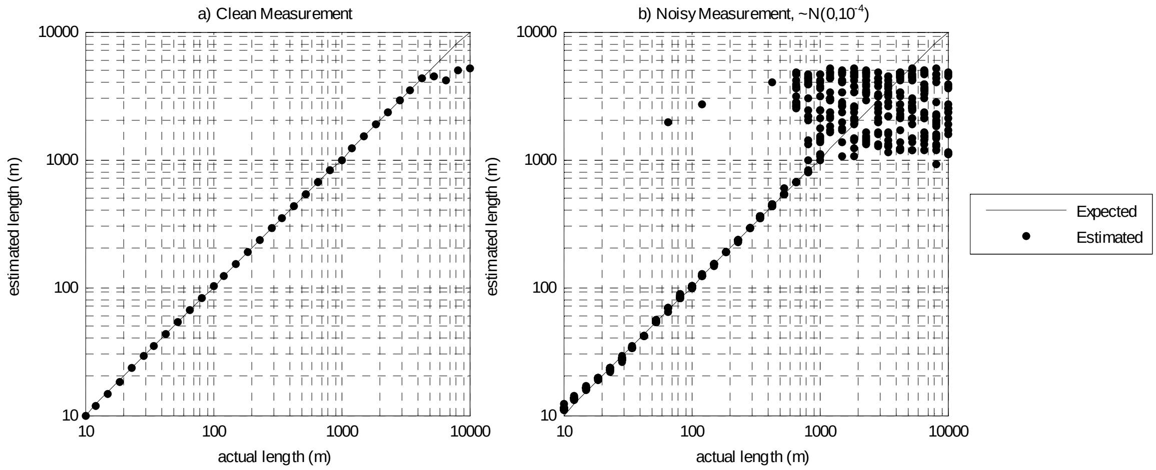

FIGURE 3. Length estimation simulation results with (a) clean and (b) noisy measurements (20 trials).

5. Conclusion

We proposed an innovative approach to estimating the length of twisted-pair segments used in POTS local loops. The procedure exploits the fact that the twisted-pair frequency response can be decomposed into a linear combination of reflection frequency responses and that these reflection frequency responses exhibit almost purely complex sinusoidal behavior. An eigen-based high resolution algorithm, the so-called MODE-Type algorithm, is utilized to separate the reflection responses. Numerical experiments indicate that the procedure can provide estimates to within 0.3% for segments from 100 m to 5 km given clean measurement data. The algorithm is found to be sensitive to measurement noise for long cable length, so that it breaks down when reflection-SNR is too low.

References

AMERICAN NATIONAL STANDARD INSTITUTES. 1999. ANSI T1.601-1999. *American National Standard for Telecommunications – Integrated Service Digital Network (ISDN) – Basic Access Interface for Use on Metallic Loops for Application on the Network Side of the NT (Layer 1 Specification).* New York: ANSI.

CEDERVALL, M., STOICA, P. & MOSES, R. 1995. 'MODE-type algorithm for estimating damped, undamped or explosive modes.' *Proc. of IEEE Asilomar Conf. on Signals, Systems, and Computers-29*, Pacific Grove, CA, vol. **2**, pp. 1101-1105.

CHENG D. K. 1989. *Field and Wave Electromagnetics*. 2nd ed. Reading, MA: Addison-Wesley.

IKUMA, T. 2001. *Model-Based Identification of POTS Local Loops for DSL Connectivity Prediction*, M.S. Thesis. Blacksburg, VA: Virginia Tech.

WAX, M. & KAILATH, T. 1985. 'Detection of signals by information theoretic criteria.' *IEEE Trans. Acoustics, Speech, and Signal Processing,* vol. **ASSP-33**, no. 2, pp. 387-392.